Measuring yields for bonds

As a general statement, the price of any financial instrument is defined as the present value of the expected cash flows from that financial instrument. In the case of bonds, there are two main components that determine the price:

1. The expected cash flow of the instrument: Coupon[1] (C) + face value (F)[2]
2. The required yield for that instrument: yield (y)

#1 is contained in the indenture of the instrument, where the basic financial terms such as its face value (or par value), maturity and coupon rate are described. #2 represents the yield or return on the capital the investor would achieve.

Mathematically:

$$P = \frac{C_1}{(1+y)} + \frac{C_2}{(1+y)^2} + \frac{C_3}{(1+y)^3} + \cdots + \frac{C_n}{(1+y)^n} + \frac{F}{(1+y)^n}$$

Where:

P: is the price of the instrument (or its present value)

C: is the coupon

F: is the face value of the bond

n: are the years to maturity

y: yield (or discount rate)

Based on the above we know that the face value of the bond and its coupon, as well as its maturity, are pre-defined values in the indenture. The bond price is also known for any investor at the time of acquiring the instrument, leaving only one unknown value in the equation, which is the discount rate at which the sum of all future cash flows equals the price of the bond. This rate is the **yield to maturity (YTM)**, which represents the overall interest rate earned by an investor who buys a bond at the market price and holds it until maturity.

Although solving for YTM can be mathematically a complex process, one can approximate YTM by using an iterative approach (trial and error), a financial calculator, an online yield to maturity calculator (such as https://iqcalculators.com/calculator/yield-to-maturity-calculator/), or Excel (i.e. RATE function).

Example of YTM calculation

To illustrate the calculation of the YTM, we can work with an example using the formula presented above. We will assume that we have a bond with a face value of $100 that pays an annual coupon of 10%, and its maturity is in 4 years. We know that the market price is $103 (i.e., it was bought at premium to the par). Below a summary of the parameters:

F: $100

[1] A coupon or coupon payment is the annual interest rate paid on a bond, expressed as a percentage of the face value and paid from issue date until maturity. Coupons are usually referred to in terms of the coupon rate (the sum of coupons paid in a year divided by the face value of the bond in question).

[2] Face value describes the nominal or dollar value of a security, as stated by its issuer. For bonds, it is the amount paid to the holder at maturity, typically in $1,000 denominations. The face value for bonds is often referred to as "par value" or simply "par."

C: 10%

n: 4

P: $103

Using the formula with an iterative approach, we can propose some values for y (tentative YTM rates) to solve it and see if how close we are to the market price of $103. Below an example using y=9.50%:

$$101.6 = \frac{10}{(1.095)} + \frac{10}{(1.095)^2} + \frac{10}{(1.095)^3} + \frac{10}{(1.095)^4} + \frac{100}{(1.095)^4}$$

With y=9.50%, the bond price P would be $101.6. This means that the yield y that equals the market price of $103 must be lower than 9.50%. We should then try with a lower rate, such as 9.20%. Below there is a summary of the results for 5 different values of y:

YTM (y)	Bond Price (P)
9.50%	101.6
9.20%	102.6
9.00%	103.2
8.80%	103.9
8.30%	105.6

Based on the above results, the YTM for a bond with market price P of $103 is higher than 9.00% but lower than 9.20%. The exact YTM using an online calculator or Excel is 9.07%.

Other measures of yield

While YTM is a widely used measure to compare bonds, it is not the only relevant yield. Many bonds have a call structure where the issuer can, at predefined prices and at a specific date, call the bond and do an anticipated repayment. For these bonds, the **yield to call (YTC)** can be calculated.

YTC is calculated in the same way as the YTM, but instead of plugging in the number of years until a bond matures, the call date and the bond's call price should be used. While there are no guarantees that a callable bond will get called, this is a risk that any investor or bondholder must consider in the calculation of its yield.

Example of YTC calculation

As mentioned above, the YTC is calculated in the same way as the YTM but using the call terms instead of the maturity terms. For illustration purposes, we can assume that we have a bond with the exact same characteristics as the one used in our prior example, but it can be called (i.e., repaid) by the issuer in 2 years at the par value, instead of waiting 4 years until its original maturity date. Then, the bond would have the exact same parameters except for n, that is reduced from 4 to 2.

Below an example using y=9.50%:

$$100.9 = \frac{10}{(1.095)} + \frac{10}{(1.095)^2} + \frac{100}{(1.095)^2}$$

With y=9.50%, the bond price P would be $100.9. This means that the yield y (in this case the YTC) that equals the market price of $103 must be lower than 9.50%. Below there is a summary of the results for 5 different values of y:

YTC (y)	Bond Price (P)
9.50%	100.9
9.20%	101.4
9.00%	101.8
8.80%	102.1
8.30%	103.0

Based on the above results, the YTC for a bond with market price P of $103 and having a call option after 2 years is close to 8.30% (the exact YTC using an online calculator or Excel is 8.31%).

For the YTC to be a risk for the investor, these two conditions should be met:

1. The bond must have the option to be called before the maturity date
2. The bond should be purchased at a premium to par value

If at least one of these conditions is not met, there is no risk that YTC<YTM.

Yield to worst (YTW) is the lower between a bond's YTM and YTC. YTW shows the worst-case scenario of profitability a bond holder may face (apart from the default of the bond issuer). For non-callable bonds, the YTW is equal to YTM.

The table below summarizes the main differences between YTM and YTW:

YTM	YTW
Calculates the yield that would be achieved if the bond were held until its maturity date.	Calculates the worst possible yield (assuming no default of the bond issuer).
It only uses the maturity date for its calculation.	Is only calculated if a bond has a call date (uses the earliest date a bond can be called)
Is calculated whether the bond is purchased at a discount or premium to par.	Is only calculated if the bond is purchased at a premium to par value.
It is not affected by a rise or decline in interest rates since the bond is being held until maturity.	Bonds are typically called if interest rates become lower than when the bond was initially issued.